Non instrusive uncertainty quantification method for models with a high number of parameters - Application to a magnetoelectric sensor

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To face the "curse of dimensionnality" met in uncertainty quantification problems when the model has a high number of random parameters, methods based on sparse approximation, like the Least Angle Regression (LAR) method, should be used. In this communication, we propose to extend the domain of applications of such methods and to apply them to quantify the impact of uncertainty on a magnetoelectric sensor performances. The sensor response is represented by a 2D finite element model with 10 random parameters. A global sensitivity analysis is carried out in order to determine the most influential parameters.

***Index terms*—Non Instrusive methods, Least Angle Regression, Uncertainty Quantification, magnetoelectric effect, finite element formulation.**

I. Introduction

Sensors based on the magneto-electric effect can measure static magnetic fields with a very high sensitivity. The design and the study of this device requires the use of a numerical model [1]. The working principle of a magnetic field sensor consists in combining magnetostrictive and piezoelectric materials [2]. These materials present some uncertain characteristics due to the manufacturing process deviations or lack of quality controls. In a previous communication [3], the influence of some of these parameters by using the Non Intrusive Spectral Projection and the Monte Carlo simulation have been investigated. These methods give similar results for a parameter number equal to 5 however the Non Intrusive Spectral Projection method was the fastest one. The application of the same method for a higher number of parameters is not straightforward due to the "curse of dimensionality" because the size of the polynomial basis to approximate the sensor response becomes high. In this communication, we propose other spectral approaches which can be applied for numerical models with more uncertain parameters. These approaches are derived from the method called Least Angle Regression method [5]; We propose to apply these approaches to a magneto electric sensor when 10 random parameters are considered. Finally we calculate the first-order Sobol indices to identify the most influential parameters.

II. Magnetic sensor

Figure 1 shows the magnetic field sensor is made with three piezoelectric with in between two magnetostrictive layers. The sensor input is the targeted static magnetic field. The output is an electric voltage v_{ac} depending on the static field. The numerical model of the magnetic field sensor is a 2D finite element model.

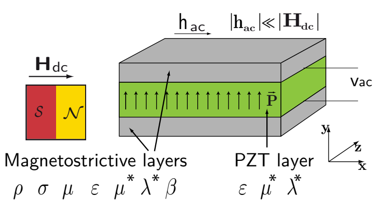

Fig. 1. Magnetoelectric sensor

The process of fabrication of such sensor is very complicated, leading to a large dispersion on the characteristics of the materials [4]. We propose in the following to study the influence of the dispersion of the material parameters on the magnitude of the electric voltage. In the stochastic approach, the uncertain parameters are modelled by random variables that we assume to be independent and uniformly distributed. We consider as random the permittivity ϵ_{mag}, the permeability μ_{mag}, the conductivity σ, the mass density ρ_{mag}, the Lamé coefficients μ^*_{mag}, λ^*_{mag} and the coupling coefficient β of magnetostrictive material. For the piezoelectric material, the permittivity ϵ_{pzt} and the Lamé coefficients μ^*_{pzt}, λ^*_{pzt} are considered as random. The parameter variations have been assumed to be of 20% on β and 5% on the other material parameters ($\mu_{mag}, \mu^*_{mag}, \lambda^*_{mag}$, ϵ_{mag}, β, σ, ϵ_{pzt}, μ^*_{pzt}, λ^*_{pzt}). In Tab I, we have reported the mean (m) and the standard deviation (SD) of each parameter.

	μ_{mag}	β	μ^*_{mag}	λ^*_{mag}	ϵ_{mag}
m	100	2.40E-5	3.85E+10	5.77E+10	5
SD	2.89	2.77E-6	1.12E+9	1.67E+9	0.6

	ρ_{mag}	σ	ϵ_{pzt}	μ^*_{pzt}	λ^*_{pzt}
m	9200	1.72E+6	15	3.85E+9	5.77E+9
SD	265	4.97E+4	0.6	1.12E+8	1.67E+8

TABLE I
Value and standard deviation of random variables

III. Least Angle Regression Method

The approach is based on the Least Angle Regression (LAR) Method which an extension has been proposed by Blatman *et al* [5] for problems with a high number of input random variables in mechanical engineering.
Let consider a numerical model $Y(\mathbf{u}(\theta))$ where $\mathbf{u}(\theta)$ is a vector of N independent uniformly distributed random variables in the interval [-1,1]. We consider a sample of S realizations of the input random variables and also the sample of the corresponding output values. We denote $\widetilde{Y}(\mathbf{u}(\theta))$ the polynomial approximation of $Y(\mathbf{u}(\theta))$.

$$Y(\mathbf{u}(\theta)) \approx \widetilde{Y}(\mathbf{u}(\theta)) = \sum_{i=1}^{P_{out}} \alpha_i \Psi_i(\mathbf{u}(\theta)) \tag{1}$$

We consider $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_{P_{out}}\}$ the polynomial basis of P_{out} terms. The value of P_{out} depends on the polynomial expansion order P and the number of input random variables N. P_{out} can be calculated by the following formula [6]:

$$P_{out} = \frac{(N+P)!}{N!P!} \tag{2}$$

We can see that the number of terms increases exponentially and can be quickly very high. For example, for a number of parameters N=10 and an order P=4, the number of polynomials is equal to $P_{out} = 1001$.

The classical non intrusive method [6] approximates the outputs by a polynomial with P_{out} terms. The LAR method reduces the computation time by selecting a small number of terms of the full polynomial basis having the greatest impact on the output. We obtain then a sparse approximation. The criterion for the selection $\Psi_i(\mathbf{u}(\theta))$ is based on the maximization of the correlation between the current residual and the predictor $\Psi_i(\mathbf{u}(\theta))$.

Once the approximation is obtained, the Sobol indices $S_{\boldsymbol{\alpha}}$ based on the decomposition of the variance are calculated in order to determine the most influential parameters $\mathbf{u}(\boldsymbol{\theta})$ on the output $Y(\mathbf{u}(\boldsymbol{\theta}))$ [7]. The sum of the Sobol indices is equal to one. All Sobol indices are positive. The first order Sobol indices S_i enables to evaluate the influence of the input $u_i(\boldsymbol{\theta})$ on the variability of the output $Y(\mathbf{u}(\boldsymbol{\theta}))$.

IV. Result

In order to evaluate the influence of the 10 input random variables on the electric voltage v_{ac} (see Fig 1), we applied the LAR method to obtain a sparse approximation of v_{ac}. Then, from this expansion, the first order Sobol indices are calculated. We have applied LAR method with an increasing number of realization S. In our case, we start to have a stable result at S=200. Table II gives the Sobol indices obtained with 200 samples.

	μ_{mag}	β	μ^*_{mag}	λ^*_{mag}	ϵ_{mag}
S_1	0	0.56	0.24	0	0

	ρ_{mag}	σ	ϵ_{pzt}	μ^*_{pzt}	λ^*_{pzt}
S_1	0.2	0	0	0	0

TABLE II
First-order Sobol indices with S=200

From the Table II, the first-order Sobol indices of the coupling coefficient β is the greatest indices, it means that the coupling coefficient β has the most impact on the electric voltage variability.

V. Improvement of LAR method

According Eq. (2), as the polynomial expansion order P or the number of random variables M increases, even by using LARs method, the computation time becomes too huge. One way to improve the LARs method is to modify the polynomial basis, so-called "primary basis" in the following, with which the LARs process is applied. The first possibility is to use a new definition of the polynomial order [5]. Assuming a weak influence of the random variable interactions on the output, a hyperbolic polynomial chaos basis is built up. For a given order, the number of terms in a hyperbolic basis is much smaller than in a classical basis where the polynomial order in classical basis is defined by the sum of the monovariate polynomial orders. Let consider a $\mathcal{A}$, a non empty finite set of indices $\boldsymbol{\alpha}$. In the classical truncation polynomial basis, the p-order set $\mathcal{A}^{M,p}$ of M random variables is given by :

$$\left|\sum_{i=1}^{M} \alpha_i\right| < p \tag{3}$$

The hyperbolic truncation proposes to determine a new set $\mathcal{A}_q^{M,p}$:

$$\left|\sum_{i=1}^{M} \alpha_i^q\right|^{1/q} < p \tag{4}$$

With q a positive number which can be arbitrary fixed. For a given value of p, if q is lower than 1, the cardinal of $\mathcal{A}_q^{M,p}$ will be lower than $\mathcal{A}^{M,p}$. The hyperbolic truncation favors the monovariate polynomials of orders lower than p and multivariate polynomials with low indices [5].

In order to reduce the number of terms of the "primary" basis, by using an error estimator, we propose a second approach based on an iterative process. At the iteration i, the LARs process is performed. An error is evaluated by using the approximations obtained from the iteration i and $i-1$. This error estimator detects the polynomials to be added to the "primary" basis of the iteration i. We jump to the next iteration $i+1$ by launching again the LARs process in this updated "primary" basis and so on. These methods have been applied successfully on academic examples. The comparison between these different approaches on the magneto electric sensor will be given in the extended version.

VI. Conclusion

In this communication, we have proposed several approaches based on the Least Angle Regression method to study a magneto electric sensor with 10 parameters. These approaches are simple to use and less time-consuming than the Monte Carlo simulation method or Non Intrusive Spectral Projection method.

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